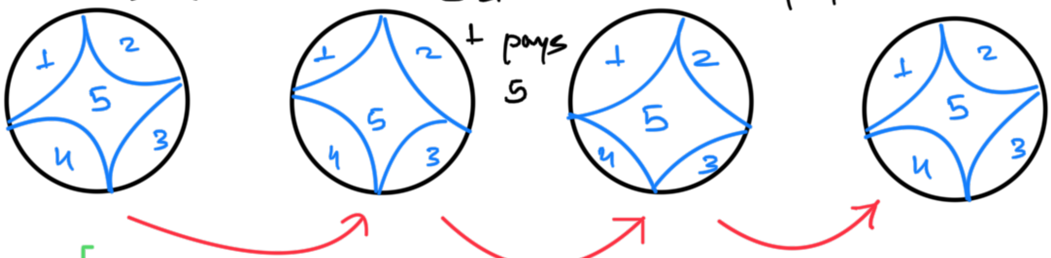
Redistributive model : Discrete.

2 lends 1 | 3 & 4 lend 1, 1 pays 5 | 1 pays back 2



$$S_1 = \begin{bmatrix} 1 & 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad S_2 = \begin{bmatrix} 0.9 & 0 & 0.2 & 0.2 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0.8 & 0 \\ 0.1 & 0 & 0 & 0 & 1 \end{bmatrix} \quad S_3 = \begin{bmatrix} 0.8 & 0 & 0 & 0 & 0 \\ 0.2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

5 gets paid for production.

1 cycle state change matrix is:

$$S = S_3 S_2 S_1$$

$$S = \begin{bmatrix} 0.72 & 0.36 & 0.16 & 0.16 & 0 \\ 0.18 & 0.59 & 0.04 & 0.04 & 0 \\ 0 & 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0.8 & 0 \\ 0.1 & 0.05 & 0 & 0 & 1 \end{bmatrix}$$

Initial wealth Distribution

$$W_0 = \begin{bmatrix} 1/10 \\ 1/10 \\ 1/10 \\ 1/10 \\ 6/10 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix}$$

$$W_1 = S W_0$$

$$W_2 = S W_1 = S^2 W_0$$

$$\vdots$$

$$W_n = S^n W_0$$

$$W_1 = \begin{bmatrix} 0.14 \\ 0.085 \\ 0.08 \\ 0.08 \\ 0.615 \end{bmatrix}$$

$$S_n = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \qquad W_n = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Notes: Context is an unproductive economy.

- 1, 2, 3, 4, 5 are people in a community.
- 5 is the machinery owner
- 1 is the businessman
- 2 is the lender
- 3 & 4 are common people who have some money but don't use it
- Eventually, after a couple hundred sales cycles, 5 has all the money & everyone else is broke.

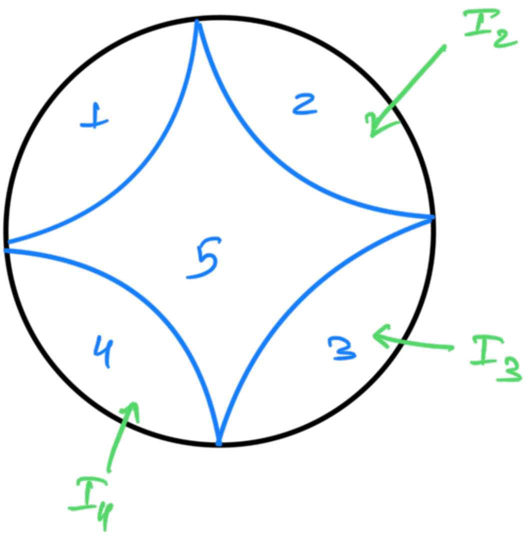
Assumptions for this economic machinery:

a) 2, 3 & 4 lack an external source of income.

b) 1 is just a seller, not a resource owner cum, seller

c) The economy is unproductive.

A more realistic representation



I_2, I_3 & I_4 are income from external sources.

Modified state change matrices:

$$S_1 \begin{bmatrix} 1 & 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad S_2 \begin{bmatrix} 0.9 & 0 & 0.2 & 0.2 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0.8 & 0 \\ 0.1 & 0 & 0 & 0 & 1 \end{bmatrix} \quad S_3 \begin{bmatrix} 0.8 & 0 & 0 & 0 & 0 \\ 0.2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Assuming ① state changes occurs annually.

② 2, 3 & 4 earn

$I_1 = 0.01$ (10% of w_0)

$I_2 = 0.02$ (20% of w_0)

$I_3 = 0.04$ (40% of w_0)

$I_5 = -0.03$ (5% of w_0 on energy costs)

$$I = \begin{bmatrix} 0 \\ 0.01 \\ 0.02 \\ 0.04 \\ -0.03 \end{bmatrix}$$

State change eq^n's:

$$w_1 = S w_0 + I$$

$$w_2 = S(S w_0 + I) + I$$

$$= s^2 w_0 + sI + I$$

$$w_3 = s^3 w_0 + s^2 I + sI + I$$

$$\vdots$$

$$w_n = s^n w_0 + \left[s^{n-1} + s^{n-2} + \dots + s + 1\right] I$$

More realistic assumptions

① Everyone has living costs as well.

② Income gradient of 2, 3 & 4 must go up at least every sale period.

$$\frac{dI}{dn} = \frac{A}{12} I \qquad A = \begin{bmatrix} 0 \\ 0.1 \\ 0.12 \\ 0.07 \\ 0 \end{bmatrix}$$

$$\ln I = \frac{A}{12} n + C$$

$$I = C e^{An/12}$$

$$I = I_0 \cdot e^{An/12}$$

$$\text{Income cap. function} = \begin{cases} e^{an} & ; \ n \le 300 \\ 1 & \end{cases}$$

③ Energy costs of 5 must go up

$$E = E_0 \cdot (1.2)^{Bn/12} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.1 \end{bmatrix}$$

$\cdot$ represents dot product

Now state change eq^ns :

$$W_1 = sW_0 + I_0 e^{A/12} - E_0 (1.2)^{B/12}$$

$$W_2 = s^2 W_0 + s I_0 e^{A/12} - s E_0 (1.2)^{B/12} + I_0 e^{2A/12} - E_0 (1.2)^{2B/12}$$

$$W_3 = s^3 W_0 + s^2 I_0 e^{3A/12} - s^2 E_0 (1.2)^{B/12} + s\left(I_0 e^{2A/12} - E_0 (1.2)^{2B/12}\right)$$
$$+ I_0 e^{3A/12} - E_0 (1.2)^{3B/12}$$

$$\vdots$$

$$W_n = s^n W_0 + s^{n-1}\left(I_0 e^{A/12} - E_0 (1.2)^{B/12}\right) + s^{n-2}\left(I_0 e^{2A/12} - E_0 (1.2)^{2B/12}\right)$$
$$+ \ldots + s\left(I_0 e^{(n-1)A/12} - E_0 (1.2)^{(n-1)B/12}\right)$$
$$+ I_0 e^{nA/12} - E_0 (1.2)^{nB/12}$$